Influence of Quantum Fluctuations on Phase Coherent Andreev Tunneling

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We study the subgap transport properties of a small capacitance normal metal-superconductor tunnel junction coupled to an external electromagnetic environment. Mesoscopic interference between the electrons in the normal metal strongly enhances the subgap conductance with decreasing bias voltage. On the other hand, quantum fluctuations of the environment destroy electronic phase coherence and suppress the subgap conductance at low bias (Coulomb blockade). The competition between charging effects and mesoscopic interference leads to a non-monotonic dependence of the differential subgap conductance on the applied bias voltage. This feature is pronounced, even if the coupling to the environment is weak and the charging energy is small.

Charge transport through a tunnel barrier between a normal metal (N) and a superconductor (S) is a widely investigated topic [1]. At energies much smaller than the superconducting gap Δ, tunneling of single particles is exponentially suppressed. Under these conditions, charge transport through an N-S interface is dominated by Andreev reflection [2]. If the normal metal and the superconductor are separated by a low transparency tunnel barrier, two-electron tunneling [3] determines the subgap conductance. Recently, there has been much interest in the subgap properties of mesoscopic N-S junctions [4,5]. The dependence of the subgap conductance of a tunnel junction on temperature or applied bias voltage can be very different, depending on the precise lay-out of the system under consideration.

If, for instance, we consider a tunnel junction between a superconductor and a thin metallic film at low temperatures, electrons move phase coherently in N and undergo multiple elastic scattering events by impurities or rough sample boundaries. As a consequence, they will be scattered back to the junction interface several times, where they attempt to tunnel into S. Two-electron tunneling involves two almost time reversed electrons. Therefore, the phase of the two-electron tunneling amplitude is not randomized by elastic scattering and the amplitudes for various tunneling attempts add up coherently. This strongly *enhances* the subgap conductance at low bias voltages [6,7]. On the other hand, in mesoscopic N-S tunnel junctions with a small junction capacitance C, charging effects [8] become important. In order to tunnel, the two electrons should overcome the characteristic Coulomb interaction energy $E_c = e^2/C$. This will strongly *suppress* the subgap conductance [9,10] at low temperatures and bias voltages $k_B T, eV \ll E_c$, a phenomenon known as Coulomb blockade of two-electron tunneling.

In the present paper we will discuss the influence of the competition between charging and interference effects on the subgap conductance of a single N-S tunnel junction.

Charging effects in a single junction are conveniently described using the so-called electromagnetic environment model [11]. In this model, electron tunneling is studied in the presence of quantum phase fluctuations due to the Johnson-Nyquist noise of the external circuit, seen by the junction. In the simplest case, this circuit consists of a capacitor C (*i.e.*, the junction capacitance) and an external series resistor $Z(\omega) = R$, see Fig. 1a. The influence of such an environment on the subgap conductance of N-S junctions has been studied before [12], but without considering mesoscopic interference. We will show how interference effects are destroyed as the series resistance R, which determines the coupling to the environment, is increased. If the charging energy is smaller than the superconducting gap, the competition between quantum fluctuations and mesoscopic interference leads to a non-monotonic dependence of the differential subgap conductance on bias voltage, even for weak coupling.

The system shown in Fig. 1a can be described by the Hamiltonian $H = H_0 + H_T$. Here, H_0 denotes the unperturbed Hamiltonian, $H_0 = H_N + H_S + H_{env}$, where H_N and H_S describe the disordered normal metal and the superconductor, respectively. The electromagnetic environment is described by the usual bosonic Caldeira-Leggett [13] Hamiltonian H_{env}. The tunnel Hamiltonian H_T transfers electrons between N and S and couples the electrons to the environment; it will be treated perturbatively. In the interaction picture H_T takes the form

$$H_T(t) = \int_N d^3 r \int_S d^3 r' \sum_\sigma [\psi^\dagger_{N,\sigma}(r,t) T(r,r') \psi_{S,\sigma}(r',t)$$
$$\times e^{-i[eVt/\hbar + \varphi(t)]} + h.c.] . \quad (1)$$

Here, $\psi_{i,\sigma}$ is a fermionic field operator for an electron with $i = N, S$ and spin $\sigma = \uparrow, \downarrow$; $T(r,r')$ is the amplitude to tunnel from a point r in N to r' in S, and V the applied bias voltage. The phase operator $\varphi(t)$ describes the voltage fluctuations at the tunnel junction, induced by the electromagnetic environment. Its dynamics is governed by H_{env}.

From an expansion in H_T up to fourth order, using standard imaginary-time techniques, one obtains the following expression for the subgap current:

$$\mathcal{I}(\omega_1, \omega_2, \omega_3) = \frac{24et_0^4}{\hbar^4} \operatorname{Im} \int_B d^2r_1 d^2r_2 d^2r_3 d^2r_4 \int_0^{\beta\hbar} d\tau_1 d\tau_2 d\tau_3 \, e^{i\omega_1\tau_1 + i\omega_2\tau_2 + i\omega_3\tau_3}$$
$$\times \mathcal{C}_N(r_1, r_2, r_3, r_4; 0, \tau_1, \tau_2, \tau_3)\Phi(0, \tau_1, \tau_2, \tau_3)\mathcal{C}_S(r_4, r_3, r_1, r_2; \tau_3, \tau_2, \tau_1, 0), \quad (2)$$

where $\beta = 1/k_B T$ and the analytical continuation for the bosonic Matsubara frequencies $i\omega_1 \to -eV/\hbar + i\delta$, $i\omega_2 \to eV/\hbar + i\delta$, $i\omega_3 \to eV/\hbar + i\delta$ has to be performed. In order to obtain (2), we assumed tunneling to occur between neighboring points on the barrier B, located at $z_B = 0$; correspondingly we put $T(r, r') = t_0\delta^3(r - r')\delta(z)$. The amplitude t_0 can be expressed in terms of the normal state conductance G_T of the tunnel barrier, $G_T = 4\pi t_0^2 S_B N_F/(\hbar v_F R_K)$. Here $R_K = 2\pi\hbar/e^2$ is the quantum resistance, N_F the density of states at the Fermi level and v_F the Fermi velocity; S_B is the area of the barrier surface. Furthermore, we introduced the four-point correlator $\mathcal{C}_i(r_1, r_2, r_3, r_4; \tau_1, \tau_2, \tau_3, \tau_4) = \langle \hat{T}_\tau \Psi_{\uparrow i}^\dagger(r_1, \tau_1)\Psi_{\downarrow i}^\dagger(r_2, \tau_2)\Psi_{\downarrow i}(r_3, \tau_3)\Psi_{\uparrow i}(r_4, \tau_4)\rangle$ describing the propagation of two electrons for $i = N, S$, as well as a four-point phase correlator $\Phi(0, \tau_1, \tau_2, \tau_3) = \langle \hat{T}_\tau e^{-i[\varphi(0) + \varphi(\tau_1) - \varphi(\tau_2) - \varphi(\tau_3)]}\rangle$ related to the voltage fluctuations. The averages $\langle\ldots\rangle$ are taken with respect to H_N, H_S, and H_{env}, respectively; in addition the correlator $\mathcal{C}$ is to be averaged over disorder.

Further simplification can be achieved following Ref. [6]. At energies much smaller than the gap Δ, two electrons propagate coherently through N over distances of the order $\xi_N = \sqrt{\hbar D/\max[eV, k_B T, \hbar/\tau_\varphi]}$, much larger than the corresponding length $\xi_S = \sqrt{\hbar D/\Delta}$ in S (D is the diffusion constant and τ_φ the phase breaking time); moreover, the lifetime $\sim \hbar/\Delta$ of a quasiparticle in S in the intermediate state is negligibly small. Therefore, we have $\mathcal{C}_S \sim \delta(r_1 - r_2)\delta(r_3 - r_4)\delta(\tau_1)\delta(\tau_2 - \tau_3)$ in Eq. (2). The dominant contribution to the subgap current can now be written as

$$I(V) = \frac{3R_K G_T^2}{\pi\hbar e S_B N_F^2}\bigg|_{i\omega_\mu \to 2eV/\hbar + i\delta}$$
$$\times \operatorname{Im}\left[\int_B d^2r \int_0^{\beta\hbar} d\tau e^{i\omega_\mu\tau}\mathcal{C}_N(r; \tau)\Phi(\tau)\right]. \quad (3)$$

The integrand of Eq. (3) is depicted diagrammatically in Fig. 1b. We see two electrons tunneling from S to N at initial position and time (0;0) thereby interacting with the environment. The electrons propagate through N to position r, where they arrive at time τ; their propagation is described by the disorder averaged two-particle correlator (Cooperon), $\mathcal{C}_N(r; \tau) = \langle \mathcal{C}_N(0, 0, r, r; 0, 0, \tau, \tau)\rangle_{\text{disorder}}$. Then they tunnel back into S, interacting once more with the environment. The wavy line in Fig. 1b denotes the phase correlator $\Phi(\tau) = \langle \hat{T}_\tau e^{-2i[\varphi(\tau) - \varphi(0)]}\rangle = \exp[4J(\tau)]$, where we choose J according to the electromagnetic environment model [11]:

$$J(\tau) = -2\int_0^\infty \frac{d\omega}{\omega}\frac{\operatorname{Re} Z_t(\omega)}{R_K}$$
$$\times \left(\coth(\beta\frac{\hbar\omega}{2})(1 - \cosh(\omega\tau)) + \sinh(\omega|\tau|)\right). \quad (4)$$

Here $Z_t(\omega) = 1/(i\omega C + 1/Z(\omega))$ is the total impedance seen by the junction.

Upon performing the analytic continuation in Eq. (3) the pair tunneling current finally is found to be

$$I(V) = \frac{3R_K G_T^2}{2\pi^2 e S_B N_F}\int_B d^2r \int_{-\infty}^\infty dE dE' \, C_N(r; E)P(E')$$
$$\times \frac{1 - \exp[-2eV\beta]}{1 - \exp[(E' - 2eV)\beta]}(f[(E - 2eV + E')/2] - f[(E + 2eV - E')/2]). \quad (5)$$

The function $C_N(r; E)$ is the spatial fourier transform of the real part of the diffusion propagator $1/[-iE + \hbar DQ^2 + \hbar/\tau_\varphi]$. The probability $P(E)$ to emit or absorb a photon with frequency $E/\hbar$ during tunneling is defined as

$$P(E) = \frac{1}{2\pi\hbar}\int dt \exp[4J(t) + iEt/\hbar], \quad (6)$$

where $J(t)$ can be obtained from Eq. (4) by putting $\tau = it$ for $\tau \geq 0$.

As an example, we will study the Andreev current (5) of a N-S tunnel junction, consisting of a quasi one-dimensional (1D) normal metal wire in contact with a superconductor via a tunnel barrier with dimensions much smaller then ξ_N. For a quasi 1D wire with cross section S_W, we have

$$\int_B d^2r C_N(r, E) = \frac{S_B}{S_W}\frac{\cos[\arctan(E\tau_\varphi/\hbar)/2]}{2\sqrt{\hbar D}(E^2 + (\hbar/\tau_\varphi)^2)^{1/4}}$$

in Eq. (5). The junction has a capacitance C and is embedded in a purely resistive environment $Z(\omega) = R$. The total impedance is thus given by $Z_t(\omega) = 1/(i\omega C + R^{-1})$. However, the calculation of $P(E)$ can only be performed numerically in this case [14]. Therefore, in order to proceed analytically, we will use the approximation $\text{Re}[Z_t(\omega)] = R\exp(-\omega/\omega_c)$, which has the correct zero-frequency limit $\text{Re}[Z_t(0)] = R$. The cut-off frequency is chosen to be $\omega_c = E_c R_K/\hbar R$, such that the approximated impedance also gives the correct phase correlation function $J(t) \sim \exp(-i2E_c t/\hbar)$ in the limit of short times. In particular, this guarantees that this approximation yields the correct behavior for $R = \infty$, namely the complete suppression of tunneling for $eV < E_c = e^2/C$.

At zero temperature the phase correlation function is readily found to be $\Phi(t) = 1/[1 + i\omega_c t]^\alpha$; its fourier transform (6) yields the probability distribution

$$P(E) = \frac{e^{-E/\hbar\omega_c}}{\Gamma(\alpha)\hbar\omega_c}\left(\frac{E}{\hbar\omega_c}\right)^{\alpha-1}\theta(E). \qquad (7)$$

The negative part of the spectrum is truncated, since at $T = 0$ photons can only be emitted. The power $\alpha = 8R/R_K$ can be interpreted as a parameter determining the coupling strength between the electronic phase and the environment.

For the non-interacting system, $\alpha = 0$, the subgap conductance is proportional to the "coherence resistance" $R_{coh} = \xi_N/\sigma S_W$ [6], where σ denotes the conductivity of the wire. In the limit of strong coupling, $\alpha = \infty$, a gap appears in the I-V curve below E_c. We will focus on the case of small charging energies $E_c \ll \Delta$. For a finite coupling and finite phase coherence time τ_φ [15], two regimes exist (see Fig. 2): (i) For very small voltages $eV \ll \hbar/\tau_\varphi$, interference is cut off by τ_φ and the I-V curve shows a power law behavior $I \propto V^{(\alpha+1)}$. This is what one would expect for noninterfering electrons. (ii) For higher voltages $eV \gg \hbar/\tau_\varphi$ the coherence length is voltage dependent and the I-V characteristic changes: $I \propto V^{\alpha+1/2}$. Remarkably, if $\tau_\varphi = \infty$ and $\alpha = \alpha_c = 1/2$, suppression of the current by charging effects and enhancement by interference cancel each other exactly at voltages below E_c/e, such that the I-V curve is linear. For values of α larger than the "critical" coupling α_c, the power is always larger than one and a Coulomb gap starts to evolve with increasing α.

The differential conductance $G(V) = dI/dV$ is strongly suppressed at $eV < \hbar/\tau_\varphi$ for *arbitrary* $\alpha > 0$. The zero bias peak, which is the fingerprint of phase coherent Andreev tunneling, is destroyed by charging effects. Instead, the differential conductance will display a peak at finite bias (Fig. 3). Increasing the voltage on the one hand lifts the Coulomb blockade, but on the other hand decreases the coherence length. If $\alpha < \alpha_c$, the maximum in the differential conductance appears at a bias $eV \lesssim \hbar/\tau_\varphi$ and will shift to zero bias for $\tau_\varphi = \infty$. The

coupling to the quantum fluctuations of the environment is too weak to fully destroy phase coherence. Therefore coherent pair tunneling is blocked only at voltages below $\hbar/\tau_\varphi$, where interference is cut off. For $\alpha > \alpha_c$, the coupling is strong enough for charging effects to dominate the behavior. The conductance is suppressed in the entire voltage regime below E_c by quantum fluctuations, which makes τ_φ superfluous as a cutoff for the divergence of G at zero bias. The maximum appears at $eV \gtrsim E_c$ and shifts towards E_c as α is increased.

At finite temperatures, the electrons can gain energy by absorbing environmental modes. Therefore, the Coulomb blockade is gradually lifted with increasing temperature. The probability distribution $P(E)$ at small energies $|E| \ll \hbar\omega_c$ can be calculated analytically, following [16]. In the long time limit, the phase correlator reads $\Phi(t) \simeq [(\pi k_B T/\hbar\omega_c)/\sinh(\pi t k_B T/\hbar)]^\alpha$. From (6) we find

$$P(E) \simeq \frac{\exp(E/2k_B T)}{2\pi\Gamma(\alpha)\omega_c}\left[\frac{2\pi k_B T}{\hbar\omega_c}\right]^{\alpha-1}\left|\Gamma\left(\frac{\alpha}{2} + \frac{iE}{k_B T}\right)\right|^2. \qquad (8)$$

For low temperatures $k_B T \ll \hbar\omega_c$ one easily calculates the zero bias differential conductance $G(T, V = 0)$ with the help of (8). For $k_B T \ll \hbar/\tau_\varphi$ we find $G(T, V = 0) \sim T^\alpha$, whereas $G(T, V = 0) \sim T^{\alpha-1/2}$ for $k_B T \gg \hbar/\tau_\varphi$.

In order to determine $G(T, V)$ away from zero bias, the function $P(E)$ should be calculated for arbitrary E. For nonzero temperatures, this can only be done numerically. Qualitatively, one expects the Coulomb blockade to be lifted if $k_B T \gtrsim \hbar/\tau_\varphi$ for weak coupling, $\alpha < \alpha_c$, whereas for $\alpha > \alpha_c$ thermal smearing becomes relevant at temperatures $k_B T \gtrsim E_c$. As an example, the differential conductance as a function of bias voltage is sketched in Fig. 4 for $\alpha < \alpha_c$ at various temperatures .

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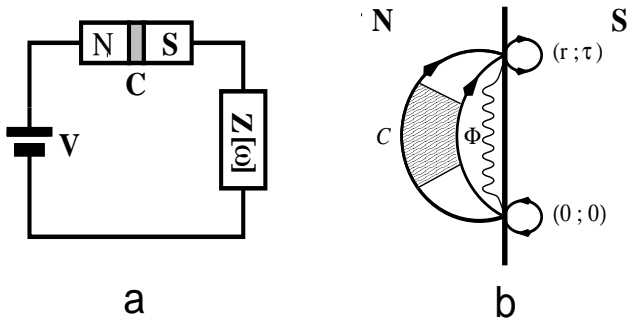
FIG. 1. (a) Single N-S junction with a capacitance C coupled to an external circuit with an impedance $Z(\omega)$ and a voltage source. (b) Two electrons propagating coherently in a disordered normal metal as a cooperon $\mathcal{C}(r;\tau)$ (half-moon) coupled to the electromagnetic environment by the phase correlator $\Phi(\tau)$ (wavy line). The upper loop describes two electrons which immediately form a Cooper pair after entering the superconductor; the lower loop describes the corresponding time-reversed process.
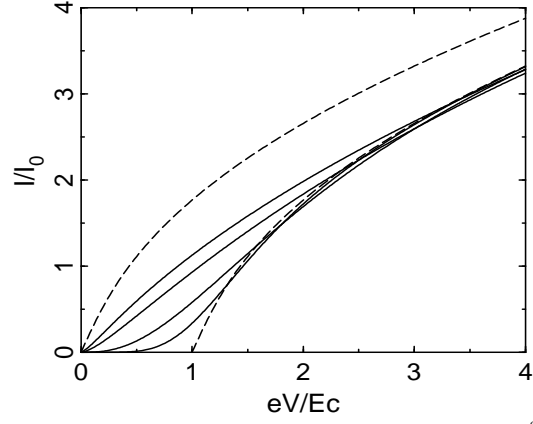

FIG. 2. Andreev current in units of $I_0 = (3G_T^2/2\pi\sigma S_W e)$ $\times\sqrt{\hbar D E_c}$ for $T = 0$ and $\hbar/\tau_\varphi = 0.5 E_c$. Curves from top to bottom correspond to $\alpha = 0$ (dashed line), $\alpha = 1/4$, $1/2$, 2, 8 (solid lines) and $\alpha = \infty$ (dashed line).

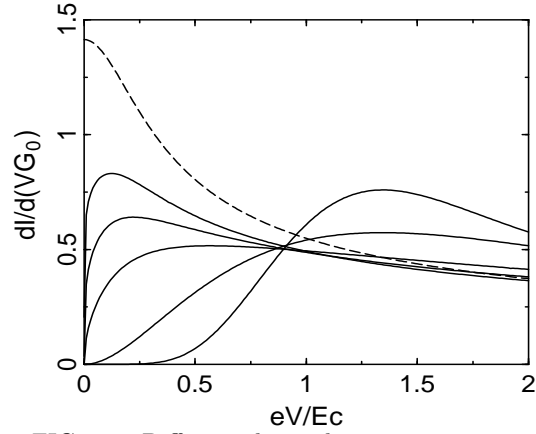FIG. 3. Differential conductance in units of $G_0 = (3G_T^2/2\pi\sigma S_W)\sqrt{(\hbar D/E_c)}$ for $T = 0$ and $\hbar/\tau_\varphi = 0.5 E_c$. The maximum evolves to the right as α is increased from $\alpha = 0$ (dashed line), taking $\alpha = 1/8$, $1/4$, $1/2$, 2, 8 (solid lines).

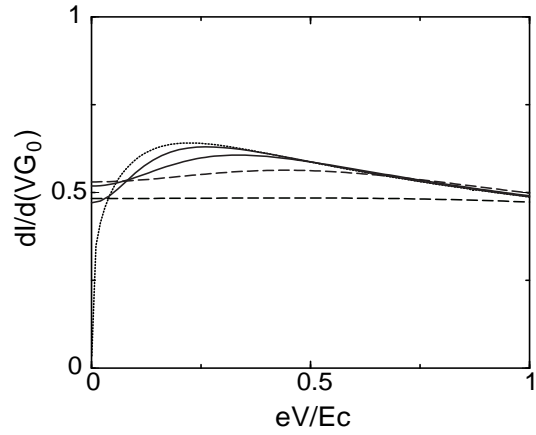FIG. 4. Sketch of differential conductance for $\alpha < \alpha_c$. From top to bottom, temperature increases: $T = 0$ (dotted line), $k_B T \ll \hbar/\tau_\varphi$ (solid lines), and $k_B T \lesssim \hbar/\tau_\varphi$ (dashed lines).